PARTICLE CHARACTERIZATION BY IMAGE ANALYSIS

Occhio's image analysis approach to particulate materials characterization is the most accurate ever imagined. Discover why this brings a real revolution to the particle science by using simple and affordable concepts.

Nothing here should appear as a black box or an obscure theory. If you have any doubts about the subject, feel free to ask Occhio's specialists. They will take the time and the adequate terms to introduce you to the fascinating world of particle image analysis.

The following paragraphs present a generic introduction to the field of particle characterization written by Prof. Eric PIRARD. We recommend you read this to better understand the "dos and don'ts" in particle characterization and more specifically in image analysis.
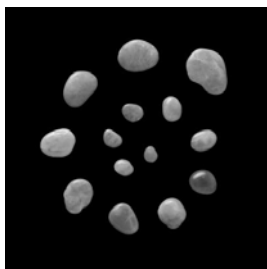
I. ESSENTIAL TERMINOLOGY

First of all, let us clarify a few terms of statistics that might be useful in the course of this reading.

My Population

The population is the whole batch of material that you want to characterize.
This can be typically the content of a bag, of a lorry, or the entire production of the day, etc.
Analysing the whole population is unaffordable and it is clearly not required!

My Sample

A sample or a sub-sample is a reasonable amount of material that can be submitted to the analysis. It is from the results of analysing the sample that conclusions will have to be drawn about the whole population. It is therefore important that the sample be "representative".

Obviously, the larger the sample, the more accurate the characterization will be. But, at some point, increasing the number of analysed particles will not contribute to a better understanding and a more accurate measurement. Every operator should optimize his sampling strategy in order to ascertain that his results are accurate enough without spending useless time and energy.

Spitefully there is no immediate answer to the question of how large a sample should be. This depends on the characteristics of the product and on the aims of the analysis.

As our intuition tells us, the more homogeneous the product, the smaller the representative sample might be… but, as Pierre Gy will tell you in the introduction of his reference textbook on sampling (ref. $) : "we should never judge beforehand the homogeneous character of an unknown material".

In terms of sampling and analytical accuracy, these are clearly two very different questions:
- *What is the average size of the particles in my product?*
- *Can I certify that no particle is larger than 460 μm?*

To answer the second question, the sample size will have to be much larger than to answer the first one!

This being said, one should realize that image analysis of particles is extremely powerful compared to most analytical techniques. Where statisticians would advise to have at least 50 particles being measured to estimate the average size of a homogeneous product (or in other terms obeying the so-called normal distribution), image analysis will easily bring tens of thousands of measurements within the lapse of a few minutes.

A good way to apprehend the difference between analysing samples of different sizes is to visualize how the distribution of the measured property evolves with the number of particles being analysed.

My particle

A particulate system is a set of individual fragments of material called particles in a generic way. It is on these fragments that individual measurements will be performed to characterize the three major attributes that fully define it:
- SIZE
- SHAPE

It is important to realize that, up to now, no technique is capable of determining independently any of these three attributes. In other words any technique gives an output that is a more or less complex combination of size, shape and nature. It is our responsibility not to misinterpret analytical results.

A simple and very familiar example is given by sieving (or sifting). The output of a sieving operation is presented as a PSD (particle size distribution) by plotting the weight of material retained in a given sieve as a function of the mesh size. Obviously this weight depends on the nature (density) of the particle and the probability of being retained by a square mesh is clearly dependent upon the shape of the particle. The more elongated the longer it will take for a particle to find its way through the mesh!

II. QUANTITATIVE PARTICLE CHARACTERIZATION

Image analysis is the first ever technique to be able to quantify any of the two attributes of a particle separately. It is essential to understand how this opens new perspectives in the science of particulate systems characterization well beyond the basic interpretation of a particle size distribution curve!

Particle Size

The word size is part of our everyday vocabulary. However, if we think about it, we will discover that its exact meaning is quite different from one field to the other and that we use it often in a confusing manner.

Let's take a few examples:
- *Speaking about the <u>size of a person</u>, we will in general refer to his tallness*
- *Speaking about the <u>size of a tennis ball</u>, we will mention its diameter.*
- *Speaking about the <u>size of a book</u>, we will mention the number of pages or in other terms its thickness.*
- *Speaking about the <u>size of a sheet of paper</u>, we will mention its length vs. width (A4, letter, etc.)*

Persons, balls, sheets or books are somehow particles (parts of a larger collection of similar objects). But, we do interact with them in very different ways… hence the use of different concepts behind the same word of size.

If we think about it, we will discover that exactly the same happens with solid particles. The meaning of size is dependent upon the process used to manufacture them or upon the application they are made for:
- *If we extrude a wire through an opening we will be interested in controlling its width because it may vary with operational conditions.*
- *if we cut the wire into fibres we will be interested in controlling the length of these fibres because this is what the cutting process is designed for.*
- *if we produce a calibrated sand by screening it, we will mention the particle width because this is the most critical parameter for a particle to pass through a mesh.*
- *if we use particles for coating a surface (e.g. thermal spray coating) or if we intend to dissolve them, we will most probably refer to their thickness because this is the dimension that controls the time to achieve complete fusion or dissolution.*

At this point we realize that, with the exception of the sphere (ball), no definition of size is truly independent of the notion of shape. But, we can already point out that size is a dimensional measure, whereas shape is clearly adimensional.
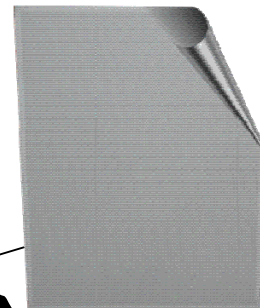
When we quote the two dimensions of a sheet of paper, we both want to qualify how large it is and how elongated the (supposedly) rectangular sheet can be.
In other words it would be equivalent to:

State two sizes:
width = 21 cm; height = 29,7 cm

State a size and a shape index:
height = 29,7 cm; elongation = 21/29,7= 0,707

From a purely mathematical point of view, it is good to refer to the notion of size as a Lebesgue measure :

- for a 1D object, the size is the length

- in 2D the size is the area

- in 3D the size is the volume

It is not very convenient to express the size of a particle in µm³, but it is important to realize that this is the only size measure, truly independent of the shape of the particle.

In other words, unless we have a 3D sensing technique, one should be aware that the output of any sizing technique is affected by the shape of the particles!

This is not a major issue if we are able to physically understand how the size measure is affected by the particle shape, but obviously in several widely used techniques this is a black box or at least a very complex relationship.

Particle Shape

The word shape is used in everyday life to express some additional characteristics of an object. Because we have no way to quantify the morphology, we make use of a large range of analogies to give indications about the shape of an object. In most cases, we will refer to simple shapes such as those inventoried from basic geometrical principles (sphere, cube, disk, pyramid, …), but in more complex situations we will use a wider grammar of shapes relating to objects of everyday life (a star, a pear, a cloud, a butterfly, …).

Interestingly, in the English language two words coexist: shape and form. Though the distinction between both is a subtle one, it is very instructive to note that "form" is used to designate the global appearance, whereas "shape" is used for more detailed observations. In other words, our everyday vocabulary reveals that we can hardly introduce into the same descriptor aspects that cover a wide range of scales.

A pencil has an elongated form but this does not tell us whether it is well sharpened or blunt. We clearly need to split the morphological information into two independent parameters.
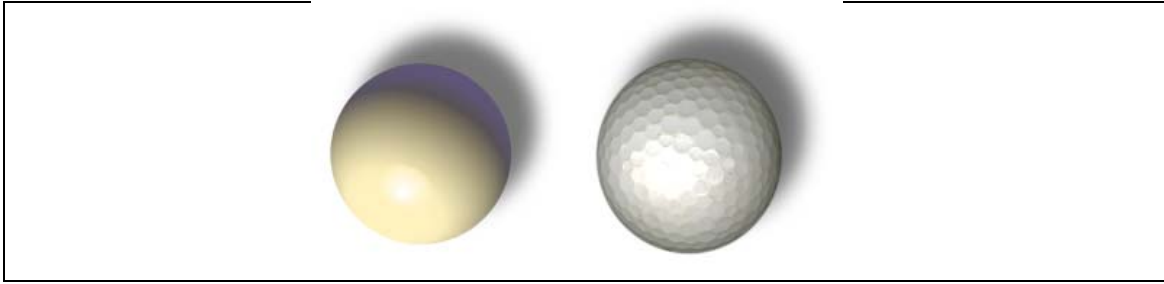

This observation is even more evident when we introduce the notion of texture. Texture is a confusing word that might also be used to qualify the internal structure of a particle, but when speaking about a surface, texture generally refers to very minute variations of the surface topography. Hence a surface can be rough, smooth, rugged, rippled, etc. even if the form remains that of a sphere!

A golf and a table tennis ball have similar size and spherical form but a different texture.

Let's keep our previous examples to add a morphological dimension :
- *Speaking about <u>a person</u>, we will add that she is thin or big, we will use a sophisticated set of mensurations for clothing purposes and we will eventually use an expressive vocabulary to be more specific (chubby, plump, skinny, scraggy,...)*
- *Speaking about <u>a tennis ball</u> we will only have to add it is spherical and tufted*
- *Speaking about <u>the book</u> we will give all three dimensions and add comments about rounded hooks or a textured cover.*

Again we notice that the way the morphology of a particle is expressed is very much dependent upon the process that is being involved.

An endless mathematical quest ...

Many mathematicians consider shape quantification as the *Quest of the Holy Graal*, and they aim to be able one day to find a unique and universal shape parameter able to discriminate any arbitrary pair of particles. Despite the fascination exerted by fractals and wavelet theories, these theories have proven to be of rather limited use in particle shape discrimination up to now.
It is probably hopeless to define a single set of shape parameters that would be of universal use, whatever the discipline. But, practical experience tells us that a multiscale approach to shape analysis is probably the most pertinent (cf. next §).

III. IMAGING PARTICLES

Considering that obtaining the digital image of a particle is central in tackling the problem of accurate size and shape analysis, it is worth understanding under which conditions and using which technology it is possible to get the best possible picture!
As has been said before, the panacea would be 3D imaging of each individual particle. But, this will not be routine for fine particle analysis before long as it remains extremely time consuming. Hopefully, we will show later in the text that the additional information this technology could contribute with respect to efficient 2D imaging is, most of the time, poorly justified.

Particle dispersion

Particle dispersion is essential for accurate analysis of particle size and shape distributions. Obviously this is not always an option when the throughput is too high or when the particles are too heavy. Specific imaging tools have been developed to roughly *estimate* the particle size distribution from images of heaps or conveyor belts. One should be aware that such technologies are very much dependent upon the quality and accuracy of the image segmentation and should only be used when no alternative can be considered such as for real time online quality control of aggregates and crushed materials.
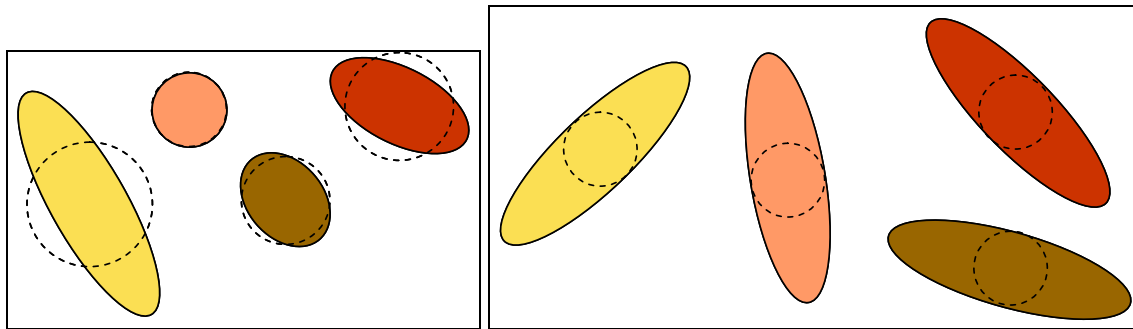
Horizontal Chute Dispersion

There is no such thing as a horizontal chute in our Newtonian world, but by horizontal chute we designate a technology that improves the vibrating feeder dispersion by combining it with the movement of a horizontal conveyor belt. Particles at the outlet of the vibrating tray fall onto a conveyor belt whose speed can be adjusted so as to stretch the granular material flow and achieve optimum dispersion. The conveyor belt can either be made of opaque material or transparent material (as patented by Occhio). This last option is the most adequate for optimal particle imaging but can be a bit tricky to operate with dirty or sticky material, requiring a very efficient conveyor cleaning step.

Advantages of the horizontal chute dispersion principle are evident:
1. Particles form a monolayer.
2. Particles are travelling at a constant and known speed in front of the camera
3. Particles are in controlled position by resting on a horizontal plane. The smallest dimension (thickness) is hidden but both the length and width are precisely measurable.
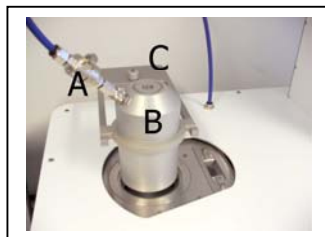
Imaging particles resting on a plane is often referred to in standards as Static Image Analysis. This is however misleading considering that with a horizontal chute the material flow is continuous and allows for imaging tens of thousands of particles a minute.

Four strictly identical particles seen under variable orientation in vertical chute (left)
or under controlled orientation in horizontal chute (right)

Vacuum Settling Chamber

Non free flowing powders with d_{50} well below 50 µm are hard to disperse mechanically. The use of wet dispersion using chemical additives (dispersants) is an option that should not be considered unless there is no alternative. For many materials efficient dispersion in air can be achieved using the vacuum dispersion device patented by Occhio. A small sample of powder is deposited onto an organic membrane closing the orifice at the top of a cylindrical vacuum chamber. The bottom of this vacuum chamber is made of a removable circular glass slide that will serve as particle support plane during the analysis. When vacuum reaches a critical level, the membrane tears open and the powder occupies the whole volume of the cylinder, gently settling onto the bottom. This principle avoids transferring mechanical forces to the particles and consequently damaging them. The final result is an excellent dispersion of a monolayer of particles resting in controlled position. When transferred under the camera an 8cm ø plate offers millions of particles to be looked at!
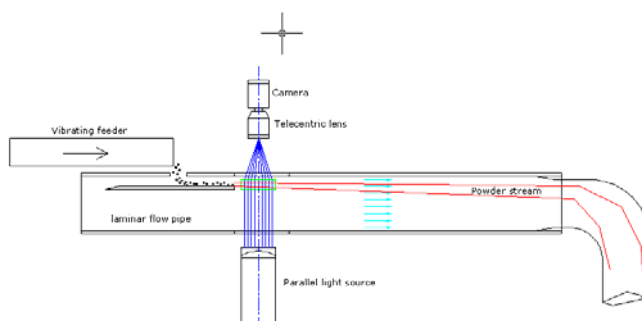
Imaging solutions in particle analysis
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A: Vacuum check valve
B: Vacuum chamber
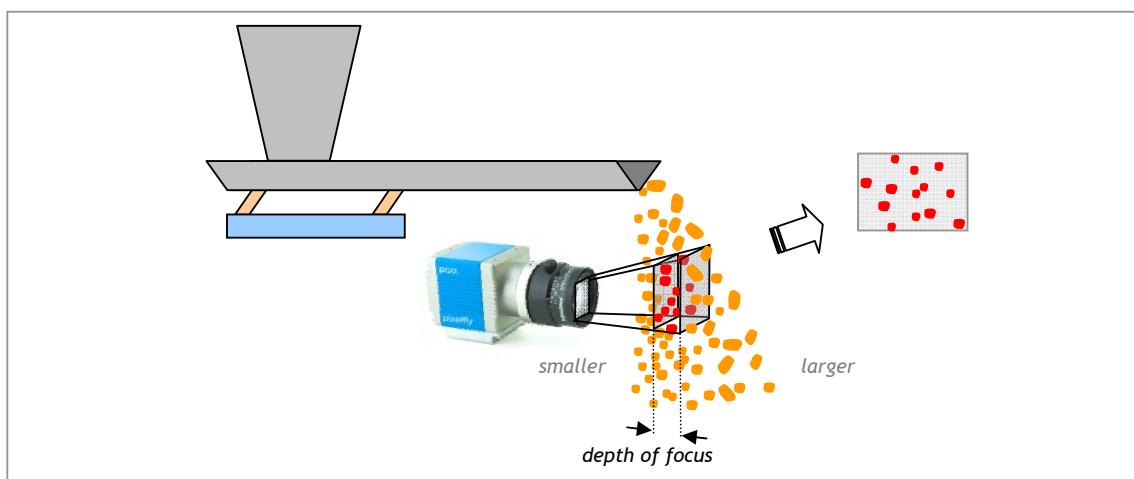C: Vacuum chamber holder
D: Sample cup
G: Glass plate

Air Flow Cell

A powerful vacuum pump whose inflow is forced between two plates in front of a camera is an alternative system when high throughput is needed. Particle orientation is fairly well controlled and particle dispersion is excellent. Instead of a relying on a delicate sampling operation to reduce the amount of material to a few tens of grams as required for the horizontal chute, this principle allows a user to analyse larger quantities of material (kg) by taking images at regular intervals. Humid, dirty and sticky materials are acceptable as feed.
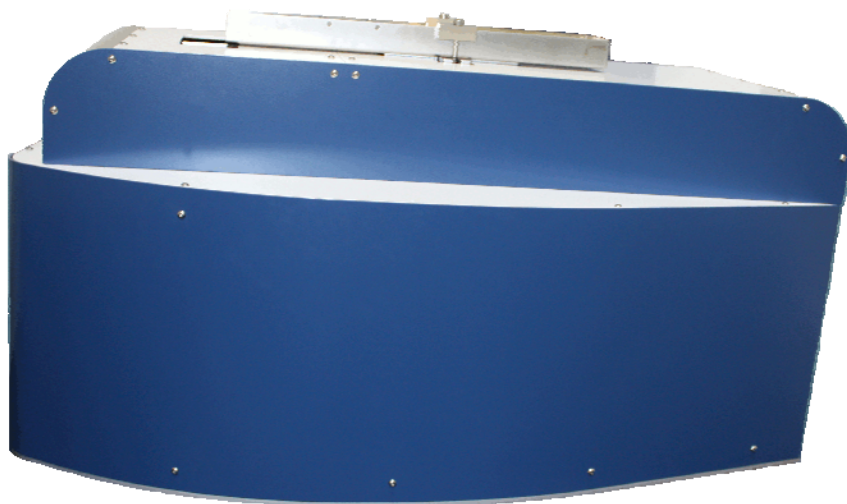

Vertical Chute Dispersion

OCCHIO is proposing a new ZEPHYR-ESR to analyse particles from 20 µm up to 3 cm

Liquid Flow Cell

For suspensions, wet powders or emulsions, the use of a flow cell is the only viable solution for particle imaging. The principle is to force a liquid to flow between two glass plates whose spacing can be adjusted to accommodate the desired particle size range. Particle orientation as well as monolayer issues should not be neglected, but considering that there is no alternative for these wet materials the flow cell is an excellent imaging option.

Morphologic parameters

Equivalent Disc Diameter

The expression of the size of a particle in terms of area units (μm^2) is not very convenient. Therefore, all systems turn it into an equivalent disc diameter (EDD or D_{Eq} or D_O) by computing the diameter of a disc that would have the same projected area (A) using the trivial formula:

$$D_O = 2.\sqrt{\frac{A}{\pi}}$$

This is the most widely used parameter, but as we will show it is by far not the most advisable method for properly sizing a particle.
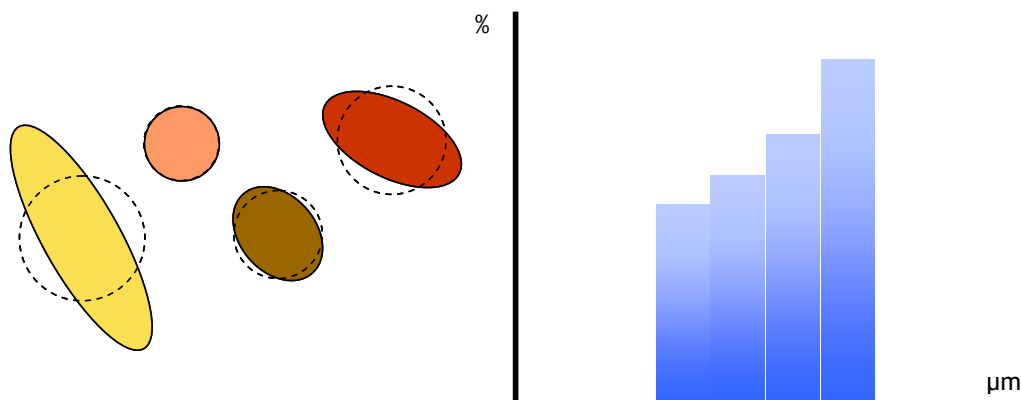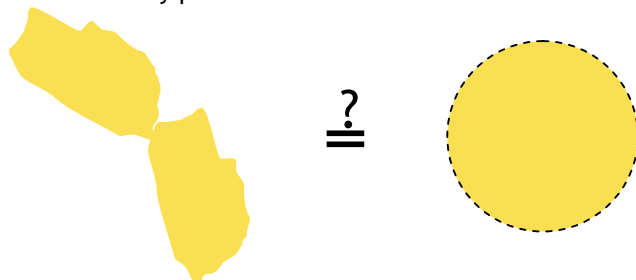Would you agree to say that these two particles have identical size?

Furthermore if you are using an imaging instrument that does not control the orientation of the particles, would you agree to consider the following size distribution as representative of your sample when all your particles are in fact strictly identical?

Finally, the equivalent disc diameter, as any parameter based on an area measurement, is poorly robust against the risk of having particles touching each other. The truth is that even the best dispersion system cannot provide a guarantee against touching particles. As an indication we should consider that at least two particles out of five hundred will appear visually (not physically!) connected. Would you agree that the result of having two connected particles is similar to having a particle double the size?

The effect will be dramatic on any particle size distribution curve!

The popularity of the Equivalent Disc Diameter is most probably due to the laziness of programmers who find it easier to sum up pixels rather than start measuring more advanced geometrical features. But, as we will try to show in the following paragraphs the computation of robust and significant parameters is the key to success in particle image analysis.
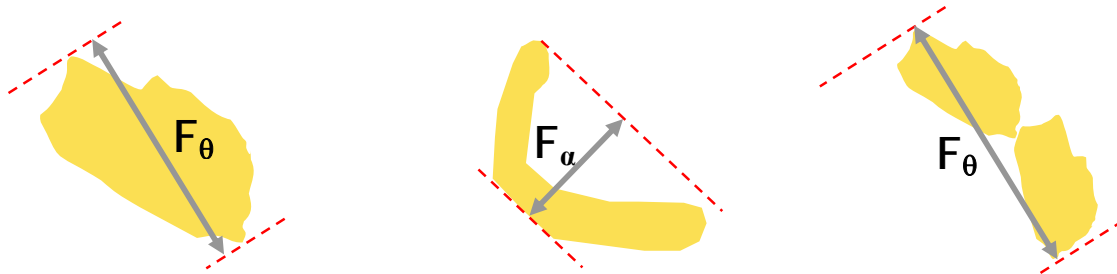
Projection Diameters

From the discrete representation of a particle it is possible to compute the length along a series of orientations. Usually 16 is considered as a minimum. This measure is called the Feret diameter or caliper diameter in most textbooks.

A clear advantage of using a Feret diameter rather than any equivalent area diameter is that this measure does correspond to a real physical dimension of the particle. However, it is best suited for compact convex particles and might be meaningless in case of tortuous or concave particle geometries. Another critical point is that it will again be extremely sensitive to touching particles! We will discuss again the Feret diameter when looking at elongation of particles.
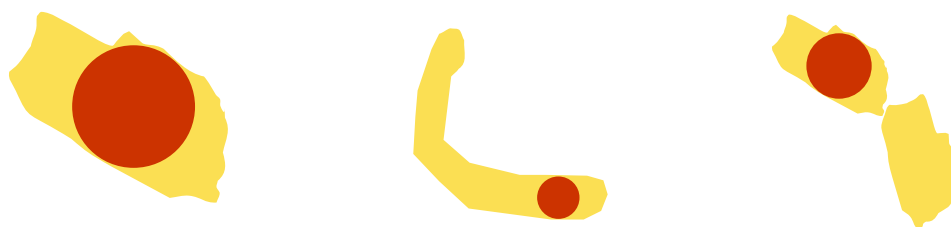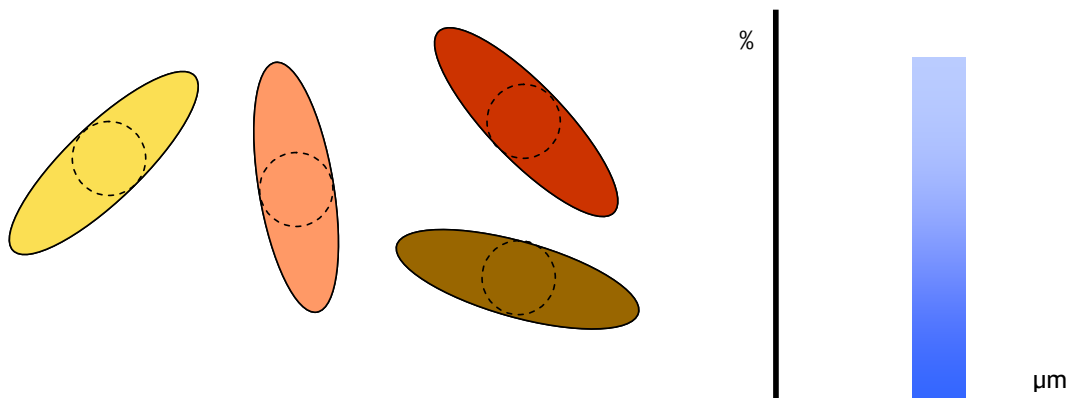
Maximum Inscribed Disc Diameter

Going one step further in the analysis of a particle outline, it is possible to compute the diameter of the maximum inscribed disc or D_{IN}.

The main arguments to use this concept is that it is a physical measure of a particle dimension, it is robust against poor particle dispersion and it does correlate perfectly with a sieving diameter when particles are looked at in a resting position (static image analysis).

By using imaging of oriented particles and the notion of maximum inscribed disc, image analysis amounts to performing a virtual sieving operation of the particles and there is no reason the numerical output should not correlate with a mechanical sieving operation (unless of course a difference in density can be suspected)

Unlike the Equivalent Disc Diameter D_O, the Maximum Inscribed Disc Diameter D_{IN} makes no assumption about the shape of the particle. In fact perfect spherical particles are very exceptional in natural as well as in synthetic products. Consider the following atomized steel particle:
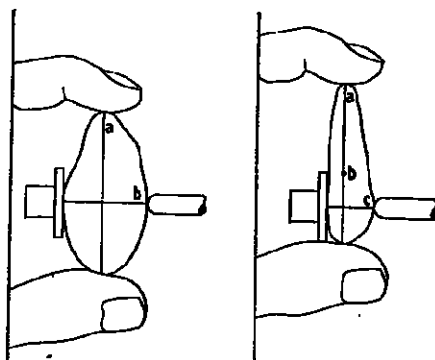

Its D_O is equal to 232.9 µm but it is capable of passing through a sieve of 209.8 µm, which makes an 11% difference. If applied to the whole product, using the D_O measure instead will significantly shift the whole particle size distribution curve to the right!

Aspect ratios

In general terms, an aspect ratio is a form (or global shape) parameter expressing the ratio between two length. The exact significance we can attribute to such a measure obviously depends upon the way the image of the particle was produced and the way the length measures were performed.

Let us first consider a three-dimensional object and clarify the concepts of length (a), width (b) and thickness (c) using Zingg's approach:

We first determine the orientation plane that gives the largest projection area of a particle. This plane contains both a and b axis orthogonal to each other. We then determine c along the direction perpendicular to the plane.

If we consider particles resting on a plane perpendicular to the optical axis as is the case in so-called static image analysis, we are in the following situation:

- Both a and b are measurable
- c is hidden and impossible to measure nor estimate
- the projected area (A) is also measurable

If we consider, as for dynamic image analysis, that particles are dispersed using a vibrating tray and falling by gravity in front of a camera, then the particles cannot be considered as perfectly oriented nor as randomly oriented. Hence:

- None of the above dimensions is truly measurable
- The largest axis is intermediate between a and b
- The smallest axis is intermediate between b and c
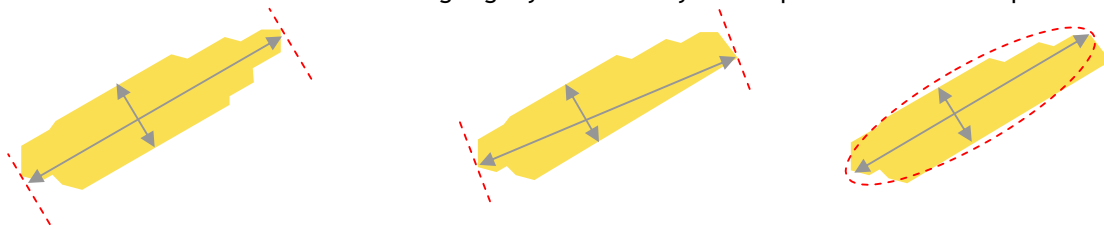- The projected area is smaller or equal to A

In terms of aspect ratios, one should understand that it is always possible to compute some kind of length and derive a ratio between any two length measures. But, if we use the terms ELONGATION (El) and FLATNESS (Fl) as in everyday language to precisely signify the following:

$$El=b/a$$
$$Fl=c/a$$

Then, only static image analysis will exactly measure Elongation (El), whereas any technique using projections of non oriented particles will be fooled by intermediate values between El and Fl.

Practical measurements of a and b can be performed using maximum and minimum Feret diameters along a series of orientations or using a faster and more robust computation method making use of Inertia moments. Consider the following slightly different crystalline particles as an example:

In the first example the maximum and the minimum Feret diameters appear to be perpendicular and there is no ambiguity in computing their ratio as the particle elongation. However in the second case (middle) the maximum and minimum Feret diameters are not perpendicular (as would be the case for a rectangle). In this case it is advisable to compute the maximum in the direction

Imaging solutions in particle analysis

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perpendicular to the minimum or to first compute an ellipse of equivalent inertia that will provide both major/minor axes orientation straightforwardly.

Learning from multiple parameters …

Because image analysis allows performing several size and shape measures on a single particle, it might be interesting to derive additional information by comparing the different values. A simple example can be given using the comparison between D_O, D_{IN} and F_{min}.
Consider the following values:
D_O= 213 µm, D_{IN}=213 µm and F_{Max}=F_{min}=213 µm   *This particle has to be perfectly round.*
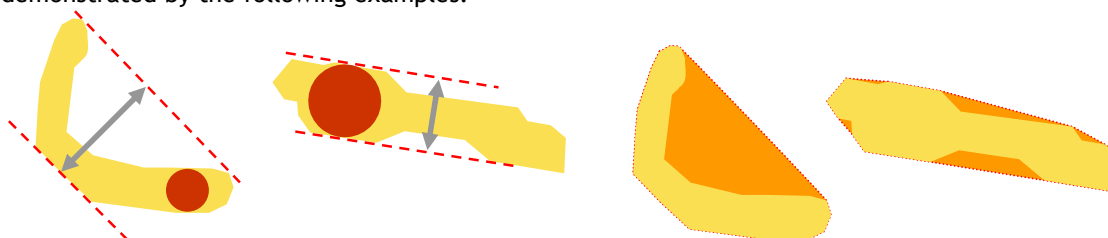D_O= 271 µm, D_{IN}=213 µm and F_{Max}=291µm F_{min}=213 µm   *This particle is elongated but not concave.*
D_O= 271 µm D_{IN}=213 µm and F_{min}=248 µm   *This particle is concave and probably elongated.*
etc.
In fact D_O is a useless parameter in this context and is best replaced by a concavity index.

Concavity Index

From the previous examples, it appears quite clearly that Feret diameters exactly correspond to the length that would be measured if we could handle the particle and bring it between the teeth of a calliper. In other words, Feret diameters are not well suited for measuring particles that are not concave. The departure between D_{IN} and F_{min} is a quick indication of concavity in a particle and we could consider using their ratio, but this does not appear to be the most sensitive method as demonstrated by the following examples:
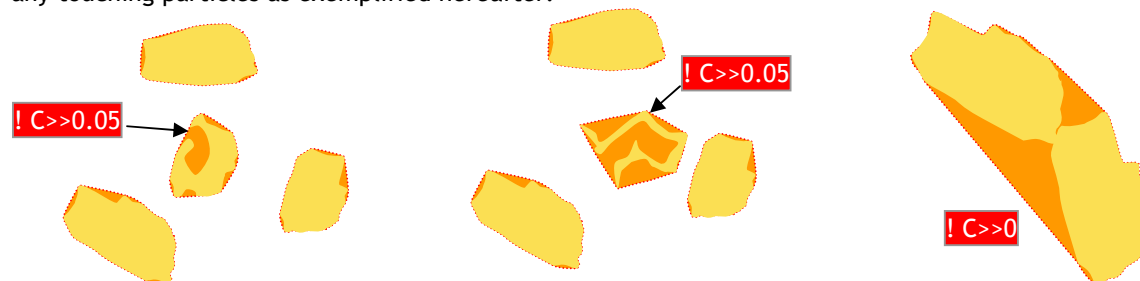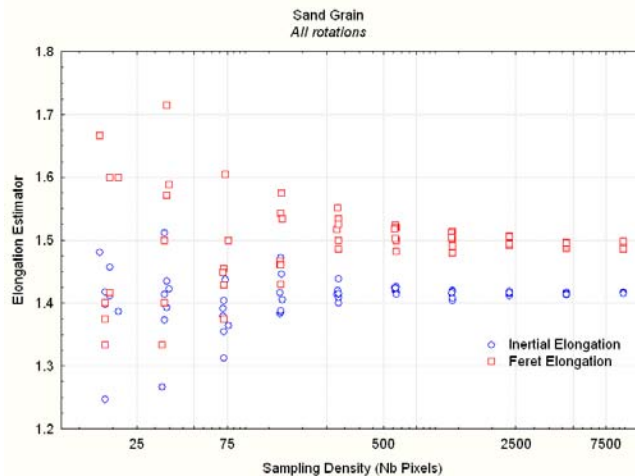

An alternative method is to draw the convex hull of the particle and express the area difference as a fraction of the convex hull area:

$$C = \frac{A_{Cvx} - A}{A_{Cvx}}$$

A value of 0 indicates a convex particle whereas anything positive suggests a certain amount of concavity. One should notice that the concavity index is just a global indicator of concavity and will not be able to make the difference between a curled shape and a shape presenting asperities. But, as such, it can be a very efficient filtering tool to remove undesired particles.
If we know by experience that a product should not have a concavity index higher than say 5%, we might very well use this threshold to remove any poorly digitized particle, any foreign material or any touching particles as exemplified hereafter:

Measuring shape regardless of the magnification ratio

We all experience the fact that the closer we look at an object the more details we will notice on its surface. In practice this means that one should be especially careful when comparing the shape of objects that have been pictured at different scales… or that are simply very different in size! This problem is too often disregarded.

A simple test will help us demonstrating where to put the limit for elongation measurements. Consider the following discrete representations of the same grain (numbers indicate area in pixels):

9000   2300   600   140

If we measure the elongation of that grain for a whole range of magnifications and rotations, we end up with the following graphic indicating much less variability at sizes above 250 pixels:

Hence, as a rule of thumb, we consider that elongation and other aspect ratio should not be considered as accurate below this magnification.

The Shape Factor

Most image analysis softwares, if not all, provide a measure of shape (or roundness) that is some sort of ratio between the area (A) and the perimeter (P) of the particle. This kind of index is very tempting because it derives from what appears to be very simple measurements and because we were told in basic geometry that

$$F = \frac{4.\pi.A}{P^2}$$

is equal to unity for a perfect disc. The fact is that it is not and that such an index suffers from major drawbacks when computed from digital images!

Before addressing the problem of proper estimation of the shape factor F let us first analyse how we can interpret its value in terms of geometry.

Consider two particles having the same area but different values of F. Obviously you will deduce from this observation that there must be a sensible difference in term of perimeter and you are right… but what causes the perimeter to increase? Is it the elongation of the particle or is it the presence of numerous asperities (roughness)?

A simple illustration of this is that the following hand-drawn shapes have strictly the same value of F=43.9%
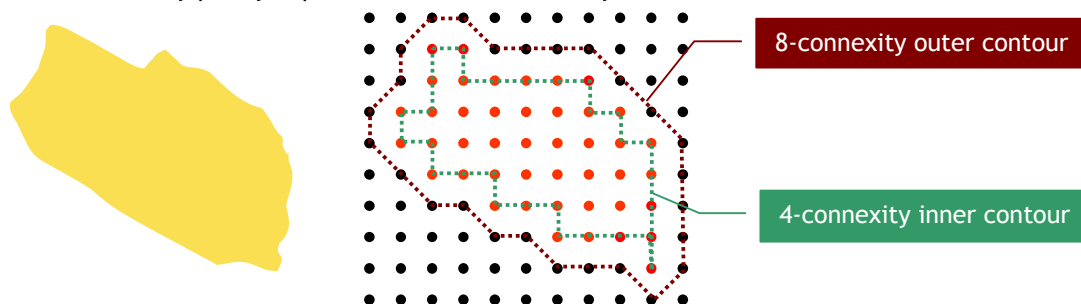

Perimeter(s)

Let us come back to the digital representation of a particle as illustrated before. Having a closer look to it, we very soon realize that by digitizing the particle we have lost any notion of interconnection between pixels. Unless we adopt a neighbourhood convention, we can hardly draw a unique boundary line uniting all the pixels. The notion of contour has been lost as is the corresponding measure of length known as perimeter… the many possible representations of contour lines are equally acceptable or unacceptable!
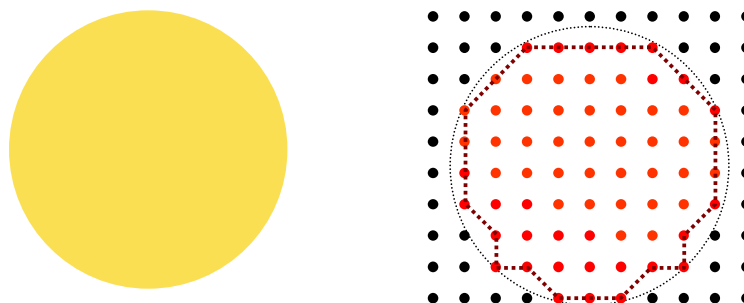
This should make us aware that unless we are sure that instruments (i.e. softwares) have adopted the same convention we can hardly compare the results of their outputs. The most adopted contour representation is probably the so-called 8-connexity inner contour, but this is hard to proof as most softwares are very poorly explicit about the choice they made.

8-connexity outer contour

4-connexity inner contour

What is true for a particle is of course true for a disc. When we draw a disc in a computer program we do tend to forget that it is made out of picture elements because the screen resolution is nowadays quite high, but if we could have a closer look to our disc, it would eventually look like this:

The reader will easily understand that the quality of the disc representation very much depends on the density of pixels (in practice the magnification of the imaging system and the size of the particle!). A careful observation of the discrete disc shown above even reveals that the disc may no longer be symmetric and that the traditional relationship between the disc radius r and the perimeter ($P = 2\pi r$) … no longer holds! This was already noted by Cauchy (17--) and later Crofton (18--) who both suggested a proper way to estimate the discrete perimeter of isometric shapes… but spitefully most if not all modern image analysers simply ignore it!

An inconvenient truth: the useless shape factor.

Let's calculate the shape factor from the discrete image shown above
Area and inner 8-c perimeter give us:
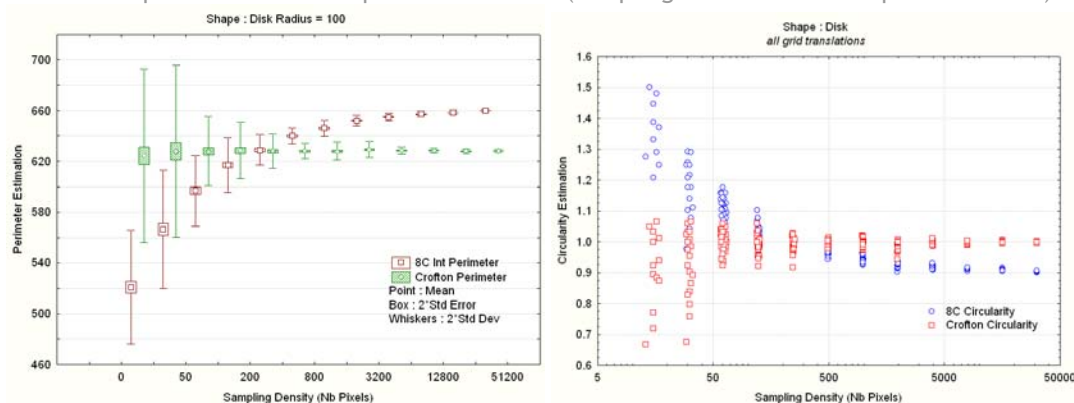
$$A = 65 \text{ pixels}$$
$$P_{8IN} = 16 + 8.\sqrt{2} = 27.31 \text{ units}$$

Hence:

$$F = 4.\pi.65 \ / \ (27.31)^2 = 1.095$$

Almost 10% more than what is theoretically possible!

The reader is invited to compute this for different perimeter representations and different pixel densities to convince himself. Alternatively, he may want to have a look at the following graphics which were produced from computer simulations (sampling discs at different pixel densities):

The graphic on the left shows the evolution of the perimeter of a disc of radius 100 units with respect to pixel densities ranging from (20 pixels/shape up to 20 000 pixels/shape). Whiskers represent the variability at a given pixel density. Clearly the 8c inner perimeter first underestimates the true value and then overestimates it, whether the Cauchy-Crofton formula shows an unbiased estimation with confidence intervals narrowing at higher densities.

The graphic on the right is similar but shows the plot of F (shape factor) values computed at different scales / pixel densities. Using Cauchy-Crofton it converges towards 1... whether with an 8c inner perimeter measure it shows values as high as 1.5 for very low pixels densities and it converges at higher pixel densities towards 0.9!

Despite the uselessness of F already demonstrated above, this has two practical consequences when dealing with real particles and conventional image analysers:

1. Smaller particles will ALWAYS show shape factors higher than larger ones and this is often simply meaningless!

2. Perfect discs have shape factors ranging from 1.5 to 0.9 ... how can we expect this measure to check the circularity of a particle or discriminate between subtle shape differences?

As a result there are more and more publications circulating around wherein people claim that the smaller particles in their product are more rounded than the larger ones when this is simply a digital illusion.
Many studies also conclude that particles depart from spheres because they show values significantly lower than 1... but ignore that even perfect spheres would give them 0.9!

There is a clear need for another tool to address particle shape analysis!

Advanced Particle Shape Analysis

The limits of traditional morphometry

Let us take a real world example to understand how far we have reached at this point in the analysis of our particles given the parameters we have defined.

Here are four synthetic diamond particles coming from the same batch. They have been pictured by placing the camera in orthogonal position with respect to their resting plane (static image analysis):

Because these particles are aimed at the superabrasives market, it is very important to be able to properly characterize their shapes and eventually predict different physical behaviours.

The following table summarizes the results using the image analysis tools that we just defined in the previous paragraphs:

	a	b	c	d
D_O (µm)	385,8	363,2	433,5	354,5
D_{IN} (µm)	455	418,6	409,5	391,3
El (%)	5,94	13,15	43,56	5,76
F (%)	74	74	61	61

Interestingly we could ask ourselves which one is the largest particle? Which one is the most elongated? Which one shows the roughest surface? Which is the most crystalline? Etc.

The largest one in terms of sieve mesh in undoubtedly (a) with a D_{IN} value of 455 µm, the largest one in terms of volume is probably (c) with a D_O of 433 µm and this is by far the most elongated one with an El=43.56% (*El is expressed here as (a-b)/(a+b)*)… but all other questions remain unsolved!

F is of little help by not being able to discriminate diamond a from diamond b and also putting into the same bin diamond c and d!

It is clear that we need more advanced concepts of shape analysis to be capable of quantifying subtle differences that we could term roughness, angularity, crystallinity, etc…

These more advanced concepts point towards the use of multiscale analysis techniques or in other words techniques that do decompose the particle outline into a series of features ranging in size from 1 pixel up to hundreds of pixels (or in practice the diameter of the particle).

The reader might be aware of such tools from the mathematical and physical literature.

Here are a few of the most popular ones:

> Fourier transform
> Fractal decomposition
> Wavelet transform

It is beyond the scope of this presentation to detail and review these techniques in the field of shape analysis. Interested readers will easily find literature on this topic. Especially on Fourier shape analysis popular in the 70's (Beddow; Luerkens;…); on Fractal analysis in the 80's (Kaye, Flook,…) and on wavelet analysis in more recent years. A brief discussion and the author's opinion on these techniques is summarized in Pirard and Hoyez,1995.

Instead of entering the world of mathematical abstractions, let us come back to our needs and try to express them in simple terms. As we will see, this will naturally draw our interest towards a theoretical framework known as mathematical morphology (Serra, 1982) that is being used intensively in image processing but is disregarded as an analytical tool.
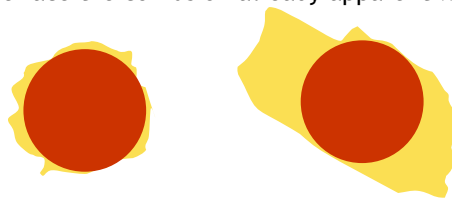
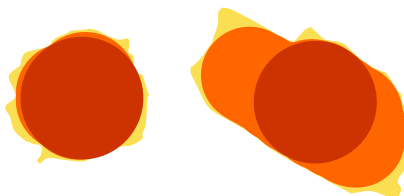*Roughness : how smooth is a particle?*

When describing the size of an irregular particle, we mentioned that the maximum inscribed disc was a pertinent measure of the particle width. It offers a strong correlation with sieving and is robust against most problems occurring during particle dispersion and image acquisition.

Let us now focus on the fraction of the surface that is not being covered by the maximum inscribed disc. This "amount" of material expresses the departure of the particle with respect to a disc (or sphere). But as is apparent from the following pair of particles, departure can be caused by something we would call "roughness" or by something we would prefer to call elongation (or non-roundness)... and here again we face the confusion already apparent with the F factor.

One way to overcome this is to combine our measure of excess material with the elongation factor suggested previously. Another way is to define a *smooth reference* by adding some tolerance to the notion of maximum inscribed disc. Let us imagine that we take the set of all inscribed discs that are at least 75% of D_{IN} in diameter:

By adding a tunable tolerance factor (γ) we can define what we will call a *global roughness* index (Rg^γ) expressing the amount of excess material still remaining after covering the shape with all discs larger of equal to $\gamma.D_{IN}$

$$Rg^\gamma = \frac{A - A(O^\gamma)}{A}$$

A definitive advantage of this kind of index with respect to the F shape factor is that it does not relate on a perimeter length measure but only on area measures, making it very robust. But, of course, it depends upon a factor γ to be chosen by the user and, as is apparent from the example, a slight change γ can induce a large change in Rg^γ. The use of Rg^γ and the best value of γ to be adopted is up to the user.

A close sensing of satellity.

An interesting application for Rg^γ, is the detection of satellites in a powder atomization process. Satellites are very minute spheres produced during atomization that tend to stick onto larger ones and produce irregular particles. Any non-spherical particle can adversely affect the flowability of the product and may even provoke very harmful powder stacking that will eventually lead to stopping the process (soldering, plasma spraying, etc.) inducing very high immobilization costs!
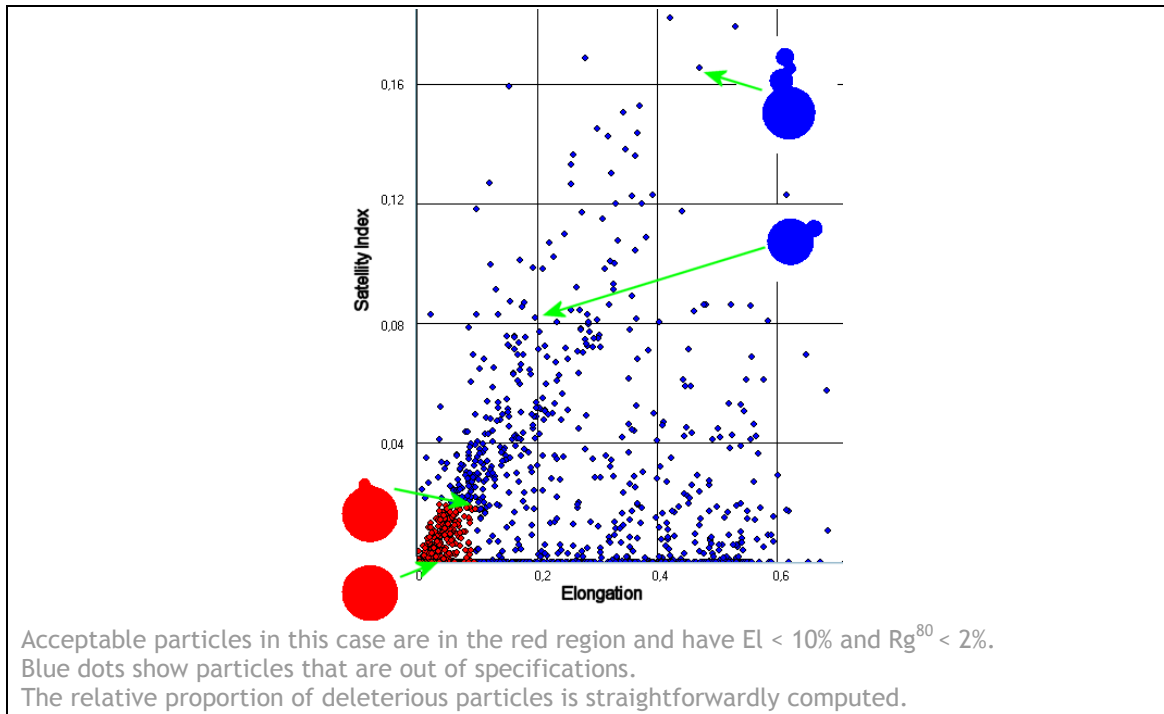
Here is a selection of atomized particles showing different degrees of satellite contamination and their corresponding measures of elongation (as (a-b)/(a+b) in %) and roughness with γ=0.8(in %):

Elongation (El)	0.78	18.64	11.71	20.78
Roughness (Rg^{80})	0.13	0.52	3.27	5.45

Interestingly the size and the elongation parameters are not sufficient to identify the occurrence of satellites, but by putting a threshold at roughness values above 1%, we will be able to pick up the deleterious particles. A statistical analysis of a complete batch of product could result in the following scatterplot of roughness versus elongation:

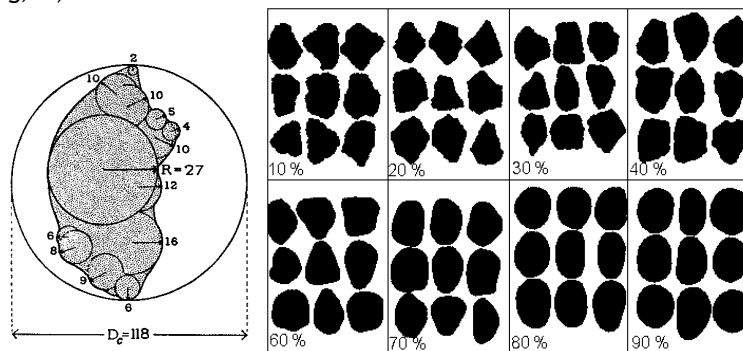Acceptable particles in this case are in the red region and have El < 10% and Rg[80] < 2%.
Blue dots show particles that are out of specifications.
The relative proportion of deleterious particles is straightforwardly computed.

Bluntness : How worn down is a particle?

The way particles wear down during abrasion is of major interest to scientist and engineers. The very first who tried to quantify this were geologists aiming at tracing back the time of residence of particles in a sedimentary system. In other words, they wanted to learn from the outer shape of a sand particle for how long it had travelled downwards a river.

Wentworth in 1919 and later Wadell observed the way cubes of rock wore down in a tumbler. From their observations, they suggested to measure the curvature of asperities and compute an average value after normalisation by the radius of the maximum inscribed disc. At that time, measures had to be done by hand… so, in order to spare precious time many authors designed visual charts inspired by those concepts. The most popular morphological chart is the one by Krumbein 1941. Since then, it has been of widespread use under many variants in various industrial and scientific sectors where shape characterization is a critical issue (mechanics, petroleum exploration, abrasive production, mining, …).

Wadell's original drawing to illustrate the average curvature of asperities and Krumbein's chart representing particles grouped into different classes of *roundness*.

What was once a tedious process of drawing discs with a compass to fit each and every asperity of a particle is nowadays a fully automatic measure that can be performed within a few milliseconds. But the step towards automation is not as easy as it may seem and it required the development of very powerful algorithms implementing new advances in discrete geometry and more specifically in mathematical morphology.
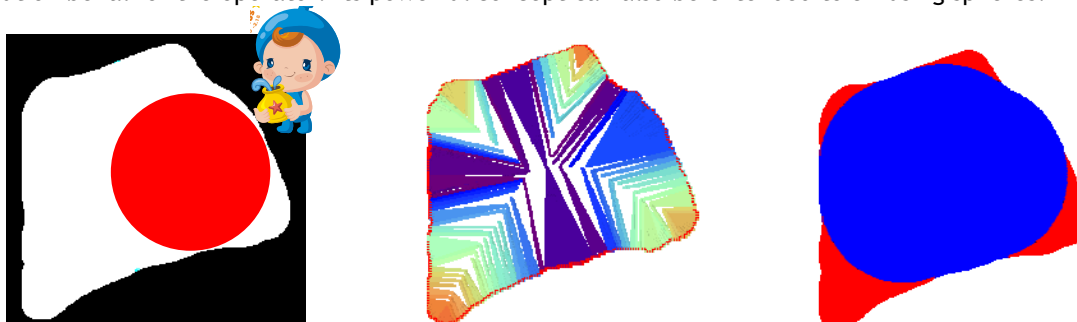
Basically, the idea is to push further the concept of filling a particle with maximum inscribed discs and do it for every point of the contour. An easy analogy is to imagine a child stepping from pixel to

pixel along the contour and blowing at each step a perfectly circular balloon until it hits another point of the contour and can't be inflated any further without being deformed.

Once this has been done for each and every pixel, we end up with a descriptor (called Calypter) giving us the centre and radius of each maximum inscribed disc. Interested readers might have noticed the similarity with the concept of skeleton and the idea of performing openings as known in mathematical morphology.

An essential property of the calypter is that its definition is unequivocal and does not require any input on behalf of the operator. Its powerful concept can also be extended to 3D using spheres.

The calypter of a shape is an extremely powerful tool that allows computing almost any desired size or shape parameter. The maximum inscribed disc (D_{IN}) and the global roughness for any tolerance γ are obvious examples, but many other dedicated concepts are now within reach.

Let us come back to our previous example of superabrasive particles and see how a bluntness index (Wv) inspired from Wadell and Krumbein's ideas could help solve the problem:

	a	b	c	d
D_{IN} (µm)	455	418,6	409,5	391,3
El (%)	5,94	13,15	43,56	5,76
Wv (%)	82.55	65.71	64.14	46.18

A value of 100% for the bluntness index can only be reached by a perfect disc whereas lower values and especially the ones below 50% indicate very irregular (rough) outlines. We clearly see from the previous table that the difference between shapes (a) and (b) that could not be clearly picked up by the previous (global) parameters are now well differentiated by the bluntness (Wv) measure. Similarly bluntness identifies a strong difference between shapes (c) and (d) that is not due to elongation but to differences in the shape of the asperities. From that perspective, shapes (b) and (c) are, not surprisingly, grouped into the same bluntness class.

Lum mean : New morphologic parameter to measure light intensity through the particles. Interesting to use to measure concentration of droplets, air bubbles and particles in emulsion.

For more information please feel free to contact OCCHIO info@occhio.be